

Rare Decays in Theories with LGS*

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Abstract

Gravitational decays in particle physics are expected to violate the B and L quantum numbers. In the standard theory, where gravity is weak because of the huge value of Planck mass, the observation of the phenomenon is far from present and probably future experimental sensitivity. On the other hand, in theories with low gravity scale, where gravity becomes stronger at shorter distances, these processes may be dangerous, predicting a too short proton lifetime. Here I discuss a possible picture of gravitational decays, which is consistent with present experimental bounds for a true gravity scale of few TeV and suggests the possibility of observing B and L violating decays with a minor improvement of the present experimental capability.

In this talk I review the picture of gravitational decays mediated by black holes I proposed in ref. [1] with Alexander Dolgov and Katherine Freese. The model is quite speculative, but predicts B and L violating processes close to the existing experimental bounds for a fundamental gravity scale of few TeV. The model could also have relevant implications in cosmology, where B violating decays are necessary to explain the observed matter-antimatter asymmetry in our universe [2].

1 Gravitational Decays

There are good arguments to believe that classical Black Holes (BHs) violate global charges such as the Baryonic (B) and the Leptonic (L) quantum numbers [3]. So, if we include gravitational interactions in particle physics, we can expect the possibility of B and L violating decays mediated by tiny BHs. The idea was indeed put forward in 1976 by Zeldovich [4] and a rough estimate of the proton lifetime is the following. We take the proton as a box of side equal to its Compton wavelength $\lambda_p \sim 1/m_p$ with three point-like quarks inside and we consider the reaction

$$q + q \rightarrow \bar{q} + l, \quad (1)$$

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where q is a quark, \bar{q} an anti-quark and l a charged lepton. Here the two quarks collide and form a virtual BH, which lastly decays violating global charges but conserving energy, angular momentum and electric charge. The rate of the process (1), Γ_p , is by definition

$$\Gamma_p = \dot{n}/n = n \sigma_{BH}, \quad (2)$$

where $n \sim m_p^3$ is the quark number density inside the proton and σ_{BH} is the cross-section of the B violating process. Since the interaction arises from a dimension six operator, the amplitude has a factor $1/M_{Pl}^2$ and the cross section can be estimated as

$$\sigma_{BH} \sim m_p^2/M_{Pl}^4. \quad (3)$$

Hence, the gravitational proton decay rate is

$$\Gamma_p \sim \frac{m_p^5}{M_{Pl}^4}. \quad (4)$$

Inserting the standard Planck mass $M_{Pl} \sim 10^{19}$ GeV into eq. (4), we find that the proton lifetime is of the order of 10^{45} yr, that is more than 10 orders of magnitude above present experimental bounds [6].

2 Theories with LGS

In last few years, theories with Low Gravity Scale (LGS) have attracted a lot of interest. The simplest example is the ADD model [7]: motivated by string theory, the observable universe would be a 4-dimensional brane embedded in a $(4+n)$ -dimensional bulk, with the Standard Model particles confined to the brane, while gravity is allowed to propagate throughout the bulk. Here extra dimensions are compact and one finds that at large distances, i.e. much larger than the size of the extra dimensions, gravity is weak, because is controlled by the usual Planck mass M_{Pl} . On the other hand, at short distances, gravity becomes stronger, because it is controlled by the true gravity scale M_* which can be as low as few TeV and therefore of the same order of magnitude of the electroweak scale. Indeed, the model was originally suggested to explain the hierarchy problem in high energy physics, that is the huge discrepancy between the Planck mass and the electroweak scale. The relation between M_{Pl} and M_* is

$$M_{Pl}^2 \sim M_*^{2+n} R^n, \quad (5)$$

where R is the size of the extra dimensions. In this approach, however, the hierarchy problem is not really solved but shifted instead from the hierarchy in energies to a hierarchy in the size of the extra dimensions which are much larger than $1/\text{TeV}$ but much smaller than the 4-dimensional universe size.

If we put $M_* \sim 1$ TeV as fundamental gravity scale into eq. (4), we find a too short proton lifetime, at the level of 10^{-12} s, which is clearly inconsistent with what we observe. So, we have essentially two possibilities: *i*) we must reject theories with LGS, because of the predicted proton lifetime, and we have to require $M_* \gtrsim 10^{16}$ GeV [8] or *ii*) the probability of the formation of a BH is suppressed with respect to the Zeldovich picture. In what follows, I discuss the second and more fascinating option, reviewing the proposal of ref. [1].

3 Classical BH Conjecture

It is well known that classical BHs in 4 dimensions cannot have arbitrary large electric charge or angular momentum. Indeed, in 4 dimensions the horizon cannot be formed if [5]

$$\left(\frac{M_{BH}}{M_{Pl}}\right)^2 < \frac{Q^2}{2} + \sqrt{\frac{Q^4}{4} + J^2}, \quad (6)$$

where M_{BH} , Q and J are respectively mass, electric charge and angular momentum of BH. So, classically, tiny BHs with a mass much smaller than the Planck mass must be electrically neutral and spinless.

In ref. [1] I conjectured that something similar may hold for BHs mediating gravitational decays as well. So, I suggested that the formation of an intermediate BH is somehow a classical process: the event horizon is formed only in particle collisions (i.e. in the s -channel of a reaction, not in the t -channel) and out of positive energies (i.e. time-energy uncertainty relation cannot create a BH with mass larger than the energy of the initial particle(s)). This implies that the decay of particles much lighter than the fundamental gravity scale can be mediated only by BH devoid of any quantum number. Of course, such a condition suppresses the process and, as I show in the next section, we can have a true gravity scale as low as few TeV without contradiction with experiments. The whole picture may look very strange, but virtual BHs are not well defined objects and it is quite probable that the standard rules of quantum field theory are not applicable to gravity. In absence of a quantum theory of gravity, this is the simplest possibility, with the advantage that its predictions are numerous and close to existing bounds.

Even if we cannot reliably calculate the decay rates of these processes, we assume they can be evaluated on dimensional grounds, with numerical coefficients of order unity. So, we guess that the coupling constant of BH to two fermions is

$$g_2 \sim R_S E, \quad (7)$$

where E is the energy of all the colliding particles which make the BH in their center of mass system, that is $E = M_{BH}$, and R_S is the BH Schwarzschild gravitational radius. On the other hand, the creation of a BH in a multi-particle collision should be further suppressed, because the particles must meet in the same small volume. By dimensional arguments we can expect that the coupling constant of BH to four fermions is

$$g_4 \sim R_S^4 E. \quad (8)$$

This choice of g_4 leads to the reasonable result that in a 4-body collision the probability of BH creation is suppressed by an additional small ratio square of BH volume to the interaction volume with respect a 2-body collision.

4 Phenomenology

4.1 Leptonic and Semi-Leptonic Decays

Let us start with the muon decay $\mu^- \rightarrow e^- e^+ e^-$. In our picture, first the muon emits a virtual photon, then the photon produces an $e^+ e^-$ pair. Next the muon and the positron form a BH devoid of any quantum number. Since the BH does not respect the family

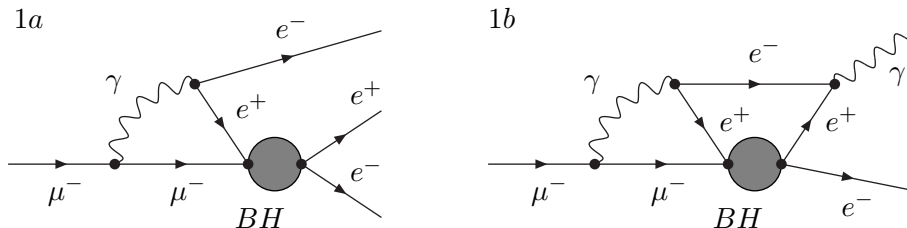


Figure 1: *a*) Muon decay $\mu \rightarrow 3e$. *b*) Muon decay $\mu \rightarrow e\gamma$.

lepton number conservation, it can decay into an e^+e^- pair, see fig. 1*a*. Using the coupling constant of BH to two fermions, we find the decay width

$$\Gamma(\mu \rightarrow 3e) = \frac{\alpha^2 m_\mu}{2^{11} \pi^5} \left(\ln \frac{M_*^2}{m_\mu^2} \right)^2 \left(\frac{m_\mu}{M_*} \right)^{4(1+\frac{1}{n+1})} \kappa^{\frac{2}{n+1}}, \quad (9)$$

where $\kappa = 0.3 - 0.5$. In the case of the ordinary (3+1)-dimensional gravity with $M_* = M_{Pl}$, the decay rate is negligibly small. On the other hand, if large extra dimensions exist, the decay could be on the verge of the experimental discovery: the present experimental constraint is [6]

$$BR(\mu^- \rightarrow e^- e^+ e^-) \Big|_{Exp} < 1.0 \cdot 10^{-12} \quad (10)$$

and requires that M_* is not smaller than 1 – 10 TeV (the exact value depends on the number of large extra dimensions n), that is around the range where M_* should be in order to explain the hierarchy problem.

Other promising and interesting reactions involving electrons and muons are the processes $e^+ + e^- \rightarrow \mu + e$ and $\mu \rightarrow e\gamma$ (see fig. 1*b*), where the predicted cross-section and branching ratio for $M_* \sim$ few TeV are surprisingly close to the current limits.

Of course, we can also consider tau decays. However, even if the tau lepton is heavier than the muon, its lifetime is shorter and the bounds on B and L violating branching ratios weaker. So, for $M_* \sim 1$ TeV and $n = 2$ one finds that the expected branching ratios of the decays $\tau \rightarrow 3l$ and $\tau \rightarrow l\gamma$ are around 10^{-11} , which surely do not contradict the existing bounds, of order of $10^{-6} - 10^{-7}$ [6]. On the other hand, τ decays with non-conservation of B and L numbers, as e.g. $\tau^- \rightarrow e^- e^+ \bar{p}$, $e^- e^- p$ and analogous ones with neutrons and neutrinos, would be strongly suppressed, because here the BH emits three quarks and one lepton (instead of two leptons): as in BH creation, multi-particle decay is suppressed due to the necessity for several particles to meet in the same small volume.

4.2 *K*-meson Decays

Good candidates for looking for L number violations are rare decays of neutral and charged *K*-mesons. Let us focus on the K^0 -meson: if two quarks constituting K^0 -meson might form BH, this BH could decay into any neutral combination of two leptons, e^+e^- , $\mu^+\mu^-$ and $\mu^\pm e^\mp$. It is easy to see that we do not contradict experimental bounds if we take $M_* > 3(4)$ TeV for $n = 2(7)$. However, it would be natural to expect that BH has the quantum numbers of the vacuum, i.e. it is a scalar object. Hence the *K*-meson, which is a

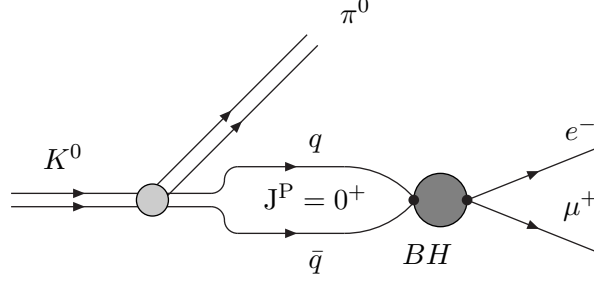


Figure 2: Kaon decay $K^0 \rightarrow \pi^0 e^- \mu^+$.

pseudoscalar, cannot transform to BH directly, but should emit some other particle in such a way that the remaining combination of the quark-antiquark system would be scalar. The simplest way is to emit a π^0 -meson, while the remainder would make a BH which would decay into $l\bar{l}$, see fig. 2. The lifetime of the decay $K \rightarrow \pi ll$ is equal to

$$\begin{aligned} \tau(K \rightarrow \pi ll) &= 0.85 \cdot 10^2 \text{ s } (g_{K\pi S} m_\pi)^{-2} \left(\frac{M_*}{\text{TeV}} \right)^{4+\frac{4}{n+1}} \left(\frac{\text{TeV}}{m_K} \right)^{\frac{4}{n+1}-\frac{4}{3}} \\ &\cdot \left(\frac{300 \text{ MeV}}{m_q} \right)^4 \frac{6.4 \cdot 10^{-3}}{f_n}, \end{aligned} \quad (11)$$

where $g_{K\pi S}$ is the coupling constant of K and π to the scalar state of quark-antiquark pair and f_n is related to integration over phase space

$$f_n = \int_\mu^{(1+\mu^2)/2} dx \sqrt{x^2 - \mu^2} (1 + \mu^2 - 2x)^{1+\frac{2}{n+1}}. \quad (12)$$

Here $\mu = m_\pi/m_K$. The factor $6.4 \cdot 10^{-3}/f_n$ is equal to 1 for $n = 2$, to 0.82 for $n = 3$, and to 0.58 for $n = 7$. In any case, for M_* at the level of few TeV we still continue to predict branching ratios close to existing bounds. So, if BHs are only scalar objects and parity is conserved, there are some interesting features/signatures: *i*) the dominant anomalous decay mode is 3 body, *ii*) the charge of the emitted pion is the same as the charge of the initial K , *iii*) the probabilities of the decays with charged and neutral leptons in the final states are approximately the same. The rather large magnitude of the branching ratios of these anomalous decays of K -mesons make them very interesting/promising candidates in the search for non-conservation of global L quantum numbers.

4.3 Proton Decay

As for the proton decay, now we need a 4-body collision in order to create an electrically neutral, colorless and non-rotating BH and the probability of the process is strongly suppressed, see fig. 3. Indeed one finds that the lifetime of the proton with respect to the inclusive decay $p \rightarrow l^+ l^- l^+$ is

$$\tau_p \approx 10^{29} \text{ yr } \left(\frac{M_*}{\text{TeV}} \right)^{10+\frac{10}{n+1}} \left(\frac{\text{TeV}}{m_p} \right)^{\frac{10}{n+1}-\frac{10}{3}} \left(\frac{100 \text{ MeV}}{\Lambda} \right)^6 \ln^{-2} (M_*/\text{TeV}) f_p^{-1}(n), \quad (13)$$

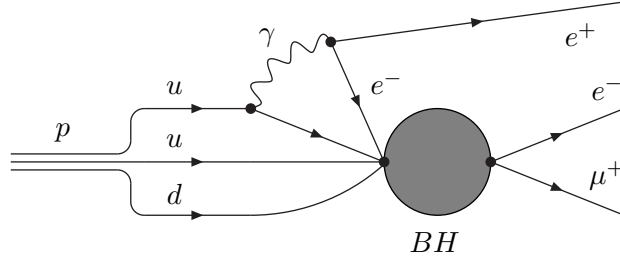


Figure 3: Proton decay.

where $\Lambda \sim 100$ MeV is the QCD scale (basically the inverse proton size) and $f_p(n)$ a numerical factor equal to 1, 1.3 and 2.2 for $n = 2, 3$ and 7 respectively. The best experimental lower bounds, at the level of $\tau_p > 10^{33}$ yr [6], are established for the modes $p \rightarrow e^+ \pi^0$ and $p \rightarrow \nu K^+$. For all other 2-body and some 3-body modes the bounds are at the level of 10^{32} yr. So, if we believe that a BH cannot go into a pseudoscalar particle, the dominant decay modes are $p \rightarrow l^+ l^+ l^-$ ($l = e, \mu$). The experimental bounds, at the level $(5 - 8) \cdot 10^{32}$ yr, are consistent with the theoretical model if the fundamental gravity scale M_* is slightly larger than 2 (8) TeV for 2 (7) large extra dimensions.

4.4 $n - \bar{n}$ Oscillation

A process where non-conservation of baryons is actively studied by experiments is neutron-antineutron transformation. In the framework of the approach presented in ref. [1], the $n - \bar{n}$ oscillations are described by the diagram of fig. 4. A rough estimate of the time of neutron-antineutron oscillations is

$$\tau_{n\bar{n}} = \left[\frac{2\alpha}{\pi} \ln \left(\frac{M_*}{m_Z} \right) \right]^{-2} \left(\frac{M_*}{\Lambda} \right)^{7 + \frac{8}{n+1}} \Lambda^{-1}. \quad (14)$$

For example, taking $n = 2$, $M_* \sim 1$ TeV and $\Lambda = 100$ MeV the oscillation time is about $3 \cdot 10^{19}$ s, that is 12 – 13 orders of magnitude above the existing experimental limit [6]: direct searches for $n \rightarrow \bar{n}$ processes using reactor neutrons put the upper limit $\tau_{n\bar{n}} > 8.6 \cdot 10^7$ s on the mean time of transition in vacuum, while the limit found from nuclei stability is slightly stronger, $\tau_{n\bar{n}} > 1.3 \cdot 10^8$. If the theoretical prediction of eq. (14) were true, the chances to observe $(n - \bar{n})$ -oscillations in the reasonable future are negligible.

One can obtain much more optimistic predictions if there exist supersymmetric partners of the usual particles. In this case, one of the quarks in the neutron can emit a neutralino, χ^0 , and become a squark, \tilde{q} . This \tilde{q} , together with remaining quarks, can form a neutral and spinless BH. This BH in turn may decay into two antiquarks, $2\bar{q}$, and anti-squark, $\tilde{\bar{q}}$. The latter captures χ^0 and becomes the usual antiquark, \bar{q} . This completes the transformation of three quarks into three antiquarks (see fig. 5). The estimated time of $(n - \bar{n})$ -oscillations would be

$$\tau_{n\bar{n}} \approx 3 \cdot 10^9 \text{ sec} \cdot 10^{\frac{12}{n+1} - 4} \left(\frac{100 \text{ MeV}}{\Lambda} \right)^6 \left(\frac{m_{SUSY}}{300 \text{ GeV}} \right) \left(\frac{\text{GeV}}{M_{BH}} \right)^{\frac{4}{n+1}} \left(\frac{M_*}{\text{TeV}} \right)^{\frac{4(n+2)}{n+1}}. \quad (15)$$

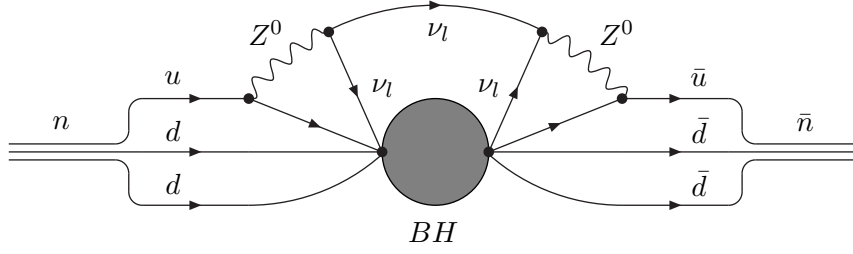


Figure 4: $(n - \bar{n})$ -oscillation.

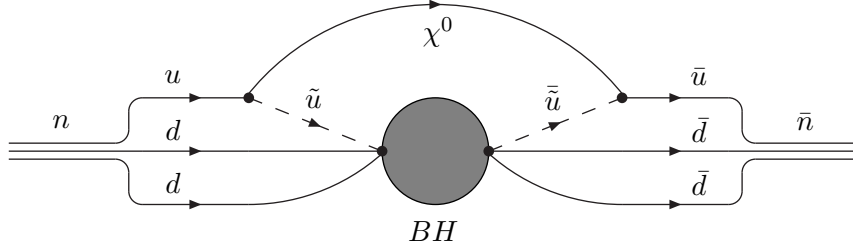


Figure 5: $(n - \bar{n})$ -oscillation with SUSY particles.

This result looks quite promising. If M_* is not too much larger than 1 TeV and the SUSY partners are not far from 300 GeV, the chances to observe neutron-antineutron transformations are very good. On the other hand, the contribution of SUSY partners to proton decay is negligible.

4.5 Summary

Table 1 presents the most promising processes for the observation of B and/or L number violation in the case of a fundamental gravity scale in TeV range. The second column of the table reports the existing experimental bounds. The third column the lower bounds on the fundamental gravity scale M_* in the case of 2 (7) large extra dimensions.

Acknowledgments

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Process	Experiment	$M_*, n = 2 (7)$
$p \rightarrow eee$	$\tau > 10^{33} \text{ yr}$	$> 2 (8)$
$\mu \rightarrow \gamma e$	$BR < 10^{-11}$	$> 1 (10)$
$\mu \rightarrow eee$	$BR < 10^{-12}$	$> 1 (10)$
$K \rightarrow \mu e$	$BR < 10^{-12}$	$> 3 (4)$
$K \rightarrow \pi \mu e$	$BR < 10^{-10}$	$> 1 (1)$
$n \leftrightarrow \bar{n}$	$\tau > 10^8 \text{ s}$	$> 1 (3) \text{ (MSSM)}$

Table 1: Summary of the most promising processes. Fundamental gravity scale M_* in TeV.

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